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NOTE ON THE AXISYMMETRIC SONIC JET

JULIAN D. COLE

1982

1. Introduction

The axisymmetric jet exhausting to sonic pressure is considered, for simplicity, under the assumptions of transonic small disturbance theory.

It is shown that the jet reaches its final state at a finite distance from the orifice. This result for the axisymmetric jet is thus the same as that for ~~the~~ a two-dimensional jet, ~~as~~ shown in Ref. 1 p. 136 ff.

Part of the argument used to show that the jet reaches its asymptotic state is local in the hodograph. ~~Thus~~ The result should also apply to a gas dynamic flow without the restriction of small disturbance theory. In the neighborhood of its final state disturbances from parallel sonic flow are in fact small.

2. Basic Equations and Boundary Value Problem

The transonic small disturbance equation for the potential can be obtained by an expansion of the following form

$$\Phi = a^*\{x + \delta\varphi(x^*,r) + \cdots\} \qquad (2.1)$$

where a^* = critical speed (flow speed for Mach number one)

δ = flow deflection angle on walls of jet (see Fig. 2.1). $\delta \ll 1$

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The axisymmetric jet exhausting to sonic pressure is considered, for simplicity, under the assumptions of transonic small disturbance theory. It is shown that the jet reaches its final state at a finite distance from the orifice. This result for the axisymmetric jet is thus the same as that for the two-dimensional jet as shown in Ref. 1 p. 136 ff. Part of the argument used to show that the jet reaches its asymptotic state is local in the hodograph. Thus, the result should also apply to a gas dynamic flow without the restriction of small disturbance theory. In the neighborhood of its final state disturbances from parallel sonic flow are in fact small.

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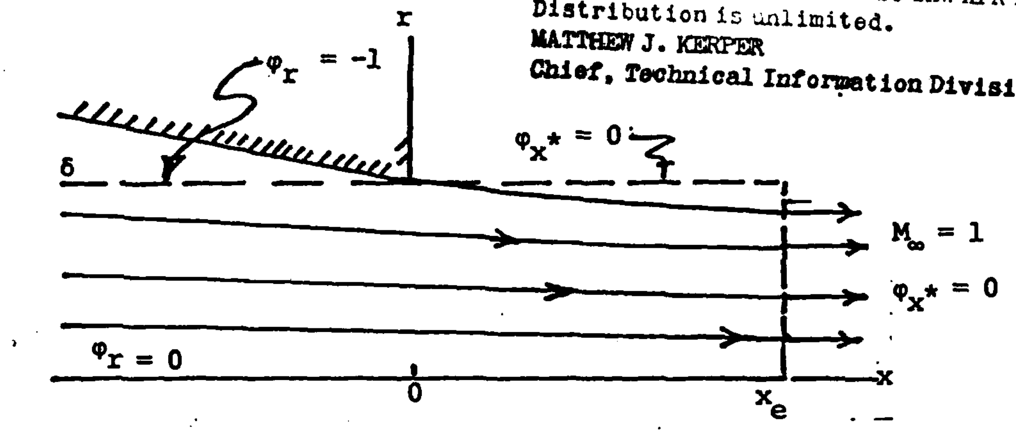

Figure 2.1. Physical Plane

The characteristic length (= 1) is the radius of the jet at its exit

$$x^* = \text{transonic coordinate} = \frac{x}{\delta^{1/3}\ (\gamma + 1)^{1/3}}\ ,\ \gamma = \text{gas constant}.$$

The resulting familiar transonic small disturbance equation is

$$\varphi_{x^*}\ \varphi_{x^*x^*} = \varphi_{rr} + \frac{1}{r}\varphi_r\ . \qquad (2.2)$$

The boundary conditions, applied as an approximation on $r = 1$, are

$$\begin{array}{lll} \text{tangent flow} & \varphi_r(x^*,1) = -1 & -\infty < x^* < 0 \\ \text{sonic pressure} & \varphi_{x^*}(x^*,1) = 0 & 0 < x^* < \infty\ . \end{array}$$

The linearized approximation to stagnation at upstream infinity has $\varphi_{x^*} \to -\infty$.

Note that the pressure coefficient (p = pressure, p^* = critical pressure, ρ^* = critical density)

$$c_p = \frac{p - p^*}{\rho^*\ a^{*2}/2} = -2\,\frac{\delta^{2/3}}{(\gamma + 1)^{1/3}}\,\varphi_{x^*}\ . \qquad (2.3)$$

3. Hodograph Equations and Problem

The structure of the flow can be seen better in the hodograph. In order to obtain a simple form some new variables are defined.

Let

$$w = \varphi_{x^*}\,, \quad \vartheta = \varphi_r\,. \tag{3.1}$$

Then, the basic equation (2.2) is equivalent to

$$\left\{\begin{array}{ll} w w_{x^*} = \vartheta_r + \dfrac{\vartheta}{r} & \text{continuity} \\ w_r = \vartheta_{x^*} & \text{irrotationality} \end{array}\right\} \tag{3.2}$$

or if $\nu = r\vartheta$

$$\left\{\begin{array}{l} r w w_{x^*} = \nu_r \\ r w_r = \nu_{x^*} \end{array}\right\} \tag{3.3}$$

and finally let $R = r^2/2$ so that

$$\left\{\begin{array}{l} w w_{x^*} = \nu_R \\ 2R w_R = \nu_{x^*} \end{array}\right\} \tag{3.4}$$

The transformation to the hodograph is carried out by

$$\begin{array}{ll} w_{x^*} = \dfrac{1}{J} R_\nu\,, & \nu_{x^*} = -\dfrac{1}{J} R_w\,. \\ w_R = -\dfrac{1}{J} x^*_\nu\,, & \nu_R = \dfrac{1}{J} x^*_w\,. \end{array} \tag{3.5}$$

Thus

$$\left\{\begin{array}{l} w R_\nu = x^*_w \\ \dfrac{1}{R} R_w = 2x^*_\nu \end{array}\right\}. \tag{3.6}$$

The basic equation for $R(w,\nu)$, an approximate Stoke's stream function, is

$$2wRR_{\nu\nu} - R_{ww} + \frac{R_w^2}{R} = 0 . \qquad (3.7)$$

Once $R(w,\nu)$ is known x^* can be calculated from (3.6).

Although the hodograph equation is also non-linear this representation has the advantage that the final state $(w = \nu = 0)$ is located at a point. A picture of the hodograph with boundary conditions and a sketch of the approximate streamlines appears in Fig. 3.1.

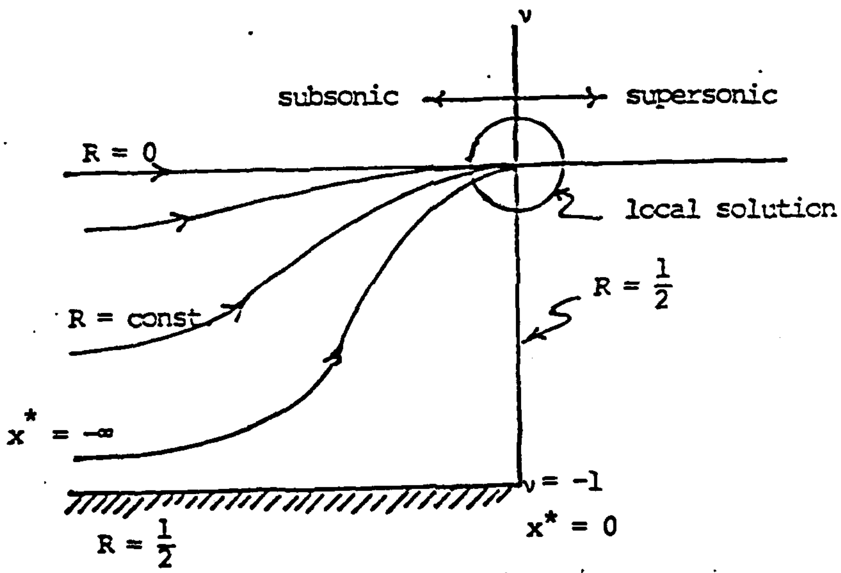

Figure 3.1. Hodograph

The origin, representing the final state, is a singular point into which all streamlines flow.

4. Local Similarity Solution

In the neighborhood of the origin of (w,ν) plane we can expect a local similarity solution. It is clear from the boundary conditions that the local solution is homogeneous of degree zero and thus has the form

$$R(w,\nu) = F(\eta) , \qquad \eta = \frac{w}{(3\nu/2)^{2/3}} . \tag{4.1}$$

The similarity coordinates $\eta = \text{const.}$ and the boundary conditions are illustrated in Fig. 4.1.

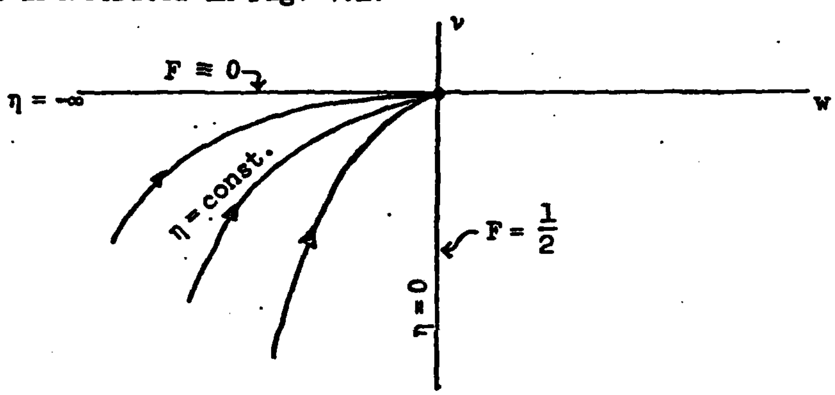

Figure 4.1. Local Hodograph

The equation for $R(w,\nu)$ (3.7) becomes

$$(2F\eta^3 - 1)\frac{d^2F}{d\eta^2} + 5\eta^2 F\frac{dF}{d\eta} + \frac{\left(\frac{dF}{d\eta}\right)^2}{F} = 0 \qquad -\infty < \eta < 0 \tag{4.2}$$

$$F(-\infty) = 0 , \qquad F(0) = 1/2 .$$

Equation (4.2) has the usual group property of transonic small disturbance equations and is invariant if

$$\begin{aligned} \eta &\to a\eta \\ F &\to a^{-3}F . \end{aligned} \tag{4.3}$$

Thus for each solution

$$F = f(\eta)$$

a one parameter family

$$F = a^{-3} f(a\eta) \tag{4.4}$$

is found. This group property allows the reduction of (4.2) to a first-order differential equation in a suitable phase plane. Let

$$\begin{Bmatrix} s = \eta^3 F \\ t = \eta^4 \dfrac{dF}{d\eta} \end{Bmatrix} . \tag{4.5}$$

Then

$$\frac{d\eta}{\eta} = \frac{ds}{t + 3s} \tag{4.6}$$

provides a mapping from a path in (t,s) to η and (4.2) becomes

$$\frac{dt}{ds} = \frac{t}{s} \frac{3s^2 - 4s - t}{(2s - 1)(t + 3s)} . \tag{4.7}$$

In the domain of interest $s \leq 0$ and $t \geq 0$. A sketch of the possible paths appears in Figure 4.2

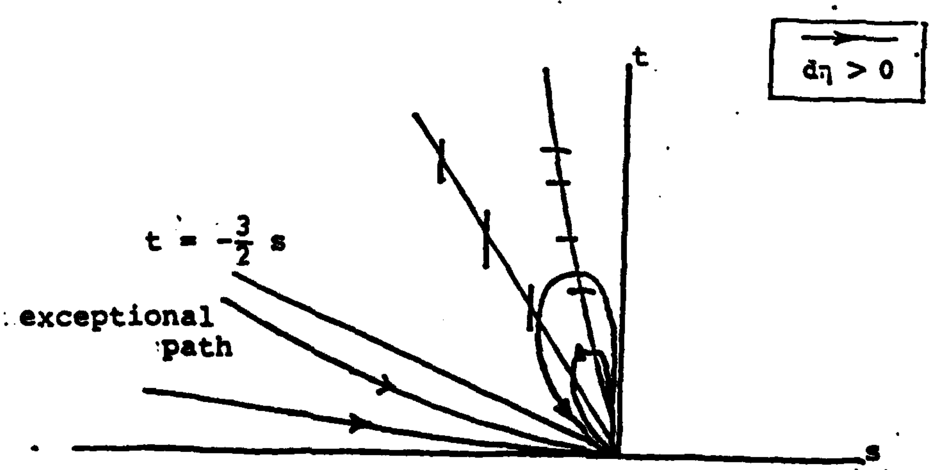

Figure 4.2. Phase Plane

The exceptional path has $t \to -\frac{3}{2}s$ as $s \to -\infty$ and

$$F(\eta) = \frac{k_\infty}{(-\eta)^{3/2}} + \cdots \, . \tag{4.8}$$

Only along this path is the boundary condition $R \to 0$ as $\eta \to -\infty$ satisfied. This solution corresponds to $R_{\nu\nu} = 0$. This path runs into the origin with $t \ll |s|$. Near the origin (4.7) is approximately

$$\frac{dt}{ds} = \frac{4}{3}\frac{t}{s}$$

so that

$$t = c_0 \, s^{4/3}$$

and

$$3\left\{\frac{1}{(1/2)^{1/3}} - \frac{1}{F^{1/3}}\right\} = c_0 \, \eta \tag{4.9}$$

showing that the boundary condition can be satisfied.

Now returning to (4.8) we note

$$R(w,\nu) = k_\infty \frac{3\,\frac{(-\nu)}{2}}{(-w)^{3/2}} + \cdots \tag{4.10}$$

and

$$x_w^* = wR_\nu = +\frac{3}{2}k_\infty \frac{1}{(-w)^{1/2}} + \cdots \qquad \text{on} \quad \eta = -\infty, \quad \nu = 0 \, . \tag{4.11}$$

Thus integration of (4.11) shows that x^* approaches a finite value as $w \to 0-$ along $\nu = 0$. The result can be checked more generally since

$$x = \nu^{1/3} \, G(\eta) + x_e^* \cdots$$

5. Remarks

The actual calculation of $x_e^* = x^*(0,0)$ demands a numerical computation, for example, of equation (3.7) for R and the use of (3.6) for x^*. The use of the local similarity solution (4.1) is helpful for these calculations. The calculations would yield the shape of the jet and the efflux.

As remarked earlier the result here is not restricted to small-disturbance flow but also applies to the full potential equation for which the flow should also approach the uniform sonic state at a finite distance from the orifice. Since x_e^* is fixed, the actual length x_e from the orifice scales as $(\gamma + 1)^{1/3}$.

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